Model Ans - MSc-I NM Paper code - 6275 - Semester Exam Dec- 2014

1.i.

x	f(x)	
0.2	0.009771222	y0
0.4	0.095476781	y1
0.6	0.393577661	y2
0.8	1.139476955	y3
1	2.718281828	y4

h	0.2
h/2*(y0+2(y1+y2+y3)+y4) =	**0.598511584**

1.ii.

$$-1/2 * \begin{matrix} 2 & -2 \\ -4 & 3 \end{matrix}$$

$$\begin{matrix} -1 & 1 \\ 2 & -3/2 \end{matrix}$$

1.iii. Refer class notes

1.iv. 6

1.v. 1≤x≤2 as f(1)=+ve and f(2)=-ve

1.vi

Let number of adults=A and children=C

$$[A \quad C]\begin{bmatrix} 20 & 15 \\ 12 & 10 \end{bmatrix} = [260 \quad 200]$$

$$[A \quad C] = \frac{1}{20}[260 \quad 200]\begin{bmatrix} 10 & -15 \\ -12 & 20 \end{bmatrix} = [(260*10+200*(-12))/20 \quad (260*(-15)+200*20)/20] = [10 \quad 5]$$

2.a.

x	y=f(x)	
0	0	y0
0.1	-0.47	y1
0.2	-0.88	y2
0.3	-1.23	y3
0.4	-1.52	y4
0.5	-1.75	y5
0.6	-1.92	y6
0.7	-2.03	y7
0.8	-2.08	y8
0.9	-2.07	y9
1	-2	y10

h	0.1
h/3*[y0+4(y1+y3+y5+y7+y9)+2(y2+y4+y6+y8)]	-1.5

2.b.

	X	y=x2+2x	X-X-bar	Y-Y-bar	(X-X-bar)*(Y-Y-bar)	sqr(X-Xbar)
	0.8	2.24	-0.2	-0.78	0.156	0.04
	0.9	2.61	-0.1	-0.41	0.041	0.01
	1	3	0	-0.02	0	0
	1.1	3.41	0.1	0.39	0.039	0.01
	1.2	3.84	0.2	0.82	0.164	0.04
	1	3.02			0.4	0.1
	X-bar	Y-bar			sum1	sum2
a1(=sum1/sum2)		**4**				
a0(=ybar-a1*xbar)		**-0.98**				
y(0.95)=		**2.82**				

3.a.

X0		1

	F(X)	F'(X)		Root	
X0		-3		4	1
X1 ->		-3		3	2
X2		2		7	1.714285714
X3	0.221574344		5.530612245		1.674222457
X4	0.00337512		5.363727763		1.673593208
X5	8.00635E-07		5.361183428		1.673593058

3.b.

4th order

	X0	Y0	h
	0	1	0.2

	K1	0.2
	K2	0.24
	K3	0.244
	K4	0.2888
Y(0.2)		**1.2428**

	x0	y0
	0	1
	0.2	1.2428

Taking h=0.2 means the error will be the order of $(0.2)^5$ which is sufficiently less.

4.a. As we know velocity, v = $\frac{dx}{dt}$ therefore, $\int dx = \int v\, dt$ where x is the distance covered by the car.

Hence, distance covered by the car in 20 sec can be written as $\int_2^{20} v\, dt$ and v=f(t) is given in the table for the required period. Here, we can use Trapezoidal rule to approximate the value of the integration taking h=2.
Distance covered = ½* [10+2(18+25+29+32+20+11+5+2)+0] = 147 km

4.b.

		x	y	z
x=	2-y/2-z/2			
y=	2-x/2-z/2			
z=	2-x/2-y/2			
		x	y	z
0th approx		0	0	0
1st approx		2	1	0.5
2nd approx		1.25	1.125	0.8125
3rd approx		1.03125	1.078125	0.9453125
4th approx		0.98828125	1.033203125	0.989257813
5th approx		0.98876953	1.010986328	1.00012207
6th approx		0.9944458	1.002716064	1.001419067
7th approx		**0.99793243**	**1.000324249**	**1.000871658**

5.a.

$Y_n(x)_1 = ((x-2)*(x-4))/((1-2)*(1-4))*(1) = x^2/3 - 2x + 8/3$

$Y_n(x)_2 = ((x-1)*(x-4))/((2-1)*(2-4))*(7) = -x^2/2 - 35/2x - 14$

$Y_n(x)_3 = ((x-1)*(x-2))/((4-1)*(4-2)) = 61/6 x^2 - 61/2x + 61/3$

$Y_n(x) = 7x^2 - 15x + 9$

$Y_n(3) = 27$

5.b.

a	0	f(a)	-1
b	1	f(b)	1
X1	0.5	f(x1)	-0.375
X2	0.75	f(x2)	0.171875
X3	0.625	f(X3)	-0.130859375
X4	0.6875	f(X4)	0.012451172
X5	0.65625	f(X5)	-0.061126709
X6	0.671875	f(X6)	-0.024829865
X7	0.6796875	f(X7)	-0.006313801

6.a.

x	f(x)	d1	d2	d3	d4
1.4	-0.216				
		0.341			
1.5	0.125		0.07		
		0.411		0.006	
1.6	0.536		0.076		0
		0.487		0.006	
1.7	1.023		0.082		
		0.569			
1.8	1.592				

h	0.1	
f'(1.4)	3.08	(=1/h(d1-1/2*d2+1/3*d3-1/4*d4)

6.b.

X0	4		

	F(X)	F'(X)	Root
X0	1	48	4
X1 ->	1	48	3.979166667
X2	0.005199291	47.50130208	3.979057211

7.

$$\left\| \begin{bmatrix} -9-\lambda & 2 & 6 \\ 5 & -\lambda & -3 \\ -16 & 4 & 11-\lambda \end{bmatrix} \right\| = 0$$

⇨ $\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$
⇨ $\lambda_1 = 1$
⇨ $\lambda_2 = -1$
⇨ $\lambda_3 = 2$

8.a

1	-1	1	1	0	0
1	-2	4	0	1	0
1	2	2	0	0	1

R2=R2-R1 =>

1	-1	1	1	0	0
0	-1	3	-1	1	0
1	2	2	0	0	1

R3=R3-R1 =>

1	-1	1	1	0	0
0	-1	3	-1	1	0
0	3	1	-1	0	1

R2=-R2

1	-1	1	1	0	0
0	1	-3	1	-1	0
0	3	1	-1	0	1

R1=R1+(-R2)

1	0	-2	2	-1	0
0	1	-3	1	-1	0
0	3	1	-1	0	1

R3=R3-3*R2

1	0	-2	2	-1	0
0	1	-3	1	-1	0
0	0	10	-4	3	1

R3=R3/10

1	0	-2	2	-1	0
0	1	-3	1	-1	0
0	0	1	-0.4	0.3	0.1

R1=R1-(-2)*R3

1	0	0	1.2	-0.4	0.2
0	1	-3	1	-1	0

0	0	1	-0.4	0.3	0.1

R2=R2-(-3)*R3

1	0	0	**1.2**	**-0.4**	**0.2**
0	1	0	**-0.2**	**-0.1**	**0.3**
0	0	1	**-0.4**	**0.3**	**0.1**

$$A^{-1} = \begin{bmatrix} 1.2 & -0.4 & 0.2 \\ -0.2 & -0.1 & 0.3 \\ -0.4 & 0.3 & 0.1 \end{bmatrix} = (-1/10) \begin{bmatrix} -12 & 4 & -2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix}$$

8.b.

Pivot elements for i and ii is -2/3

Pivot elements for i and iii is -1/3

⇨ from i and ii => -26y+5z=-21

⇨ from i and iii => -7y+28z=21

⇨ z=1

⇨ system of linear equations in upper triangular form

$$3x + y - z = 3$$
$$-26y + 5z = -21$$
$$z = 1$$

x=1, y=1, z=1